RANDOM EFFECTS IN STRUCTURES ATTACKED BY NON-STATIONARY SEISMIC EXCITATION WITH DIFFERENT DELAY IN SUPPORTS

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ABSTRACT

The paper presents an entirely non-stationary random approach to the solution of seismic response of structures, taking into account the time-delay of the seismic attack to the supports of the structure. The solution is based on the FEM discretization, the different starts of the excitations in the supports being introduced in the corresponding system of differential equations. The idea of the solution is the integral spectral decompositions of both, the excitation and the response, into the deterministic mean value variable in time, and into spectral components of the random part of the process in the form of Stiltjes integral. The solution to the deterministic component of the response and the kernel of the spectral decomposition of the response was done using the Laplace transformation in matrix form. The random component of the response is described by the matrix of cross-correlation functions, which has been obtained as a double nonstationary convolution in time.

1. INTRODUCTION

The excitation due to the natural seismicity has the character of a strongly nonstationary random process. Such an excitation evokes a strongly nonstationary response, where the transient processes of different type are combined with the forced nonstationary vibration itself. Respecting these facts the real response will be obtained, the character of which completely differs from the results of various stationary or deterministic approximations or from the results based on current seismic Codes [9,10]. Considering extensive structures with distant supports and with the dense spectrum of natural frequencies, like bridges, pipe or high-voltage lines, this discrepancy can cause a significant underestimation of seismic effects. With these structures the time delay of the start of the excitation in individual

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supports can increase or decrease the response, according to the character of the structure and of the excitation.

The authors get on with their previous works [7,8], in which the importance of the transient effects for the behaviour of the structures, excited by nonstationary random seismic movement, is obvious. The effect of time delay of the excitation in different parts of the structure is given in the present paper.

Performing the discretization for the use of FEM the problem leads to the solution of the system of ordinary differential equations with random nonstationary right-hand sides and with different time-delays of the beginning of the excitation. The coefficients of the equations are considered as constants, the damping as nonproportional due to the possibility of the use of materials with different dampings or of special damping devices.

From the methods for the solution to the statistical problem the integral spectral decompositions have been used; using it, the solution for most real seismic events can be expressed in the form of analytical expressions, which can be easily algorithmized, implemented into the existing FEM systems and used in practical calculations. An important advantage of this approach is, that it makes possible a detailed qualitative analysis of the influence of input parameters on the response of the structure.

2. *MATHEMATICAL MEAN AND CORRELATION MATRIX OF THE NONSTATIONARY RESPONSE*

Let's shortly describe a common procedure for the solution to one typical stochastic response of a discrete or discretized system. Assummig the linear system and gaussian excitation, the response is gaussian too, thus the mathematical mean value, the dispersion (variance) and mutual correlations of all components sufficiently describe the response. The equation of motion of such a system is

$$\mathbf{A}\ddot{\mathbf{u}}(t) + \mathbf{B}\dot{\mathbf{u}}(t) + \mathbf{C}\mathbf{u}(t) = -\mathbf{F}\dot{\mathbf{v}}(t) - \mathbf{G}\mathbf{v}(t) \qquad (1)$$

$\mathbf{A}, \mathbf{B}, \mathbf{C}$ – square symmetrical matrices of real numbers of the dimension $(n \times n)$, n – the number of degrees of freedom of the system. They describe the inertial and damping properties of the system and the rigidities of its internal constrains. The damping is considered as nonproportional.

$\mathbf{F}, \mathbf{G}$ – constant rectangular matrices of real numbers of the dimension $(n \times m)$, m – the number of degrees of freedom in the supports, where the kinematic excitation is being applied to the system. They describe the damping and rigidity properties of the constrains between the system and the moving supports.

$\mathbf{u}(t)$ – column vector of the length n of the response of the system

$\mathbf{v}($ – column vector of the length m which describes the movements transferred to the system through the supports. The mechanism of this transfer has been described by the matrices $\mathbf{F}, \mathbf{G}$.

The elements of the vector $\mathbf{v}(t)$ are considered as given random, continuous, gaussian nonstationary processes. Respecting the character of the seismic events it will be supposed that this excitation can be with sufficient accuracy expressed in the form

$$\mathbf{v}(t) = \mathbf{m}(t) \cdot \mathbf{v}_s(t) \tag{2}$$

$\mathbf{v}_s(t)$ – the column vector of the length m of the gaussian processes, stationary in the sense of correlations

$\mathbf{m}(t)$ – a square diagonal matrix $(m \times m)$ of deterministic modulation functions.

$$\mathbf{m}(t) = \mathrm{diag}\,|m_1(t), \ldots, m_m(t)| \tag{3}$$

The initial conditions are supposed to be homogeneous, viz.

$$\mathbf{u}(t)\Big|_{t=0} = 0\,; \qquad \dot{\mathbf{u}}(t)\Big|_{t=0} = 0 \tag{4}$$

It can be proved that the processes $\mathbf{v}(t)$ and $\mathbf{u}(t)$ can be expressed in the form of integral spectral decompositions

$$\mathbf{v}(t) = \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\,\omega\, t}\, \mathbf{m}(t)\, \mathrm{d}\mathbf{\Phi}(\omega) + \mathbf{m}(t)\mathbf{v}_{so} \tag{5}$$

$$\mathbf{u}(t) = \int_{-\infty}^{\infty} \mathbf{U}(\omega, t)\, \mathrm{d}\mathbf{\Phi}(\omega) \; + \mathbf{u}_0(t) \tag{6}$$

$\mathbf{v}_{so}, \mathbf{v}_{sc}$ – mean values resp. centered parts of processes $\mathbf{v}_s(t)$

$\mathbf{u}_0(t)$ – deterministic part (mathematical mean value) of the response

$\mathbf{U}(\omega, t)$ – $(n \times m)$ rectangular matrix of unknown deterministic functions describing transformation of random part of the excitation in frequency domain into random part of the response in time domain

$\mathbf{d\Phi}(\omega)$ – spectral differentials of the centered components of the processes $\mathbf{v}_s(t)$, taking in mind that, e.g. [6]:

$$\mathbf{E}\left\{\mathrm{d}\mathbf{\Phi}(\omega_1) \cdot \overline{\mathrm{d}\mathbf{\Phi}^t(\omega)}\right\} = \delta(\omega - \omega_1)\, \mathbf{S}_v(\omega_1)\, \mathrm{d}\omega_1\, \mathrm{d}\omega \tag{7}$$

$\mathbf{E}\{\cdot\}$ – operator of the mathematical mean

$\mathbf{S}_v(\omega)$ – square matrix $(m \times m)$ of cross-spectral densities of centered parts of the stationary processes $\mathbf{v}_s(t)$

It can be also proved ([7,8]) that the unknowns $\mathbf{u}_0(t)$, $\mathbf{U}(\omega, t)$ can be expressed using equations

$$\mathbf{A}\ddot{\mathbf{u}}_0(t) + \mathbf{B}\dot{\mathbf{u}}_0(t) + \mathbf{C}\mathbf{u}_0(t) = -(\mathbf{F}\dot{\mathbf{m}}(t) + \mathbf{G}\mathbf{m}(t))\mathbf{v}_{s0}(t) \tag{8}$$

$$\mathbf{u}_0(t)\Big|_{t=0} = 0\,; \qquad \dot{\mathbf{u}}_0(t)\Big|_{t=0} = 0 \tag{9}$$

$$\mathbf{A}\ddot{\mathbf{U}}_0(\omega,t)+\mathbf{B}\dot{\mathbf{U}}_0(\omega,t)+\mathbf{C}\mathbf{U}_0(\omega,t)=-(\mathbf{F}\dot{\mathbf{m}}(t)+(\mathrm{i}\omega\mathbf{F}+\mathbf{G})\mathbf{m}(t))\,\mathrm{e}^{\mathrm{i}\omega t} \tag{10}$$

$$\mathbf{U}_0(\omega,t)\Big|_{t=0}=0\,; \qquad \dot{\mathbf{U}}_0(\omega,t)\Big|_{t=0}=0 \tag{11}$$

It is evident that the deterministic part of the response is independent from the random part of excitation. For a majority of important cases of modulation functions $\mathbf{m}(t)$ the matrix $\mathbf{U}(\omega,t)$ and the vector $\mathbf{u}_0(t)$ can be expressed analytically, e.g. using Laplace transform, but the numerical or experimental procedures can be used too, if necessary. The solution of (10) and (11) can be obtained in the form of convolution:

$$\mathbf{U}(\omega,t)=-\sum_{l=1}^{n}\int_0^t\left(\mathbf{S}_l\,\mathrm{e}^{p_l t-(p_l-\mathrm{i}\omega)\tau}+\overline{\mathbf{S}_l}\,\mathrm{e}^{\overline{p_l}t-(\overline{p_l}-\mathrm{i}\,\omega)\tau}\right)\cdot$$
$$\cdot\,(\mathbf{F}\,\dot{\mathbf{m}}(\tau)+(\mathrm{i}\,\omega\,\mathbf{F}+\mathbf{G})\,\mathbf{m}(\tau))\,\mathrm{d}\,\tau \tag{12}$$

Taking into account the diagonal form of matrix $\mathbf{m}(t)$ in accordance with (3) and assuming the starting points for m excitation processes at $t_0=\Delta t_j\geq 0$, (12) can be rewritten as

$$\mathbf{U}(\omega,t)=-\sum_{j=1}^{m}\sum_{l=1}^{n}\int_{\Delta t_j}^{t}\left(\mathbf{S}_l\,\mathrm{e}^{p_l t-(p_l-\mathrm{i}\omega)\tau}+\overline{\mathbf{S}_l}\,\mathrm{e}^{\overline{p_l}t-(\overline{p_l}-\mathrm{i}\,\omega)\tau}\right)\cdot$$
$$\cdot\,(\mathbf{F}\,\dot{\mathbf{m}}_j(\tau)+(\mathrm{i}\,\omega\,\mathbf{F}+\mathbf{G})\,\mathbf{m}_j(\tau))\,\mathrm{d}\,\tau \tag{13}$$

Correlation matrix of the response will be obtained directly from its definition based on $\mathbf{U}(\omega,t)$ - see e.g. [1]; taking into account (6) we obtain

$$\mathbf{K}_u(t_1,t_2)=\mathbf{E}\left\{(\mathbf{u}(t_1)-\mathbf{u}_0(t_1))\,(\overline{\mathbf{u}(t_2)}-\overline{\mathbf{u}_0(t_2)})^t\right\}=$$
$$=\int_{-\infty}^{\infty}\mathbf{U}(\omega,t_1)\,\mathbf{S}_v(\omega)\,\overline{\mathbf{U}^t(\omega,t_2)}\,\mathrm{d}\,\omega \tag{14}$$

For applications the dispersions $\mathbf{D}_u(t)$ of the response $\mathbf{u}(t)$ (i.e. $t_1=t_2$) are usually the most important. Putting (13) into (14) and supposing $t_1=t_2=t$ we have:

$$\mathbf{D}_u(t)=\sum_{i,j=1}^{m}\int_{-\infty}^{\infty}\left\{\sum_{k,l=1}^{n}\int_{\Delta t_i}^{t}\int_{\Delta t_j}^{t}\left(\mathbf{S}_l\,\mathrm{e}^{p_l t-(p_l-\mathrm{i}\omega)\tau_1}+\overline{\mathbf{S}_l}\,\mathrm{e}^{\overline{p_l}t-(\overline{p_l}-\mathrm{i}\,\omega)\tau_1}\right)\cdot\right.$$
$$\left.\cdot\,\mathbf{S}_{mij}(\omega,\tau_1,\tau_2)\cdot\left(\mathbf{S}_k^t\,\mathrm{e}^{p_k t-(p_k+\mathrm{i}\omega)\tau_2}+\overline{\mathbf{S}_k^t}\,\mathrm{e}^{\overline{p_k}t-(\overline{p_k}+\mathrm{i}\,\omega)\tau_2}\right)\mathrm{d}\tau_1\,\mathrm{d}\tau_2\right\}\mathrm{d}\omega \tag{15}$$

where it has been denoted

$$\mathbf{S}_{mij}(\omega,\tau_1,\tau_2)=(\mathbf{F}_i\dot{m}_i(\tau_1)+(\mathrm{i}\omega\,\mathbf{F}_i+\mathbf{G}_i)m_i(\tau_1))\cdot S_{vij}(\omega)\,\cdot$$
$$\cdot\,(\mathbf{F}_j^t\dot{m}_j(\tau_2)+(-\mathrm{i}\,\omega\,\mathbf{F}_j^t+\mathbf{G}_j^t)m_j(\tau_2)) \tag{16}$$

and $\mathbf{F} = |\mathbf{F}_1, ..., \mathbf{F}_n|$; $\mathbf{G} = |\mathbf{G}_1, ..., \mathbf{G}_n|$.

The matrices $\mathbf{S}_l$ and numbers p_l entirely describe the system itself, having nothing to do with the excitation. It can be shown [5] that

$$\mathbf{S}_l = \mathbf{x}_l \cdot \mathbf{x}_l^t \tag{17}$$

where $\mathbf{x}_l$ is a column vector with n elements, namely the eigenvector of the matrix polynomial $\mathbf{Q}(p)$, and further

$$\mathbf{Q}(p_l)\,\mathbf{x}_l = (\mathbf{A}\,p_l^2 + \mathbf{B}\,p_l + \mathbf{C})\,\mathbf{x}_l = 0 \tag{18}$$

where following relations have been used

$$\mathbf{X} \cdot \overline{\mathbf{X}}^t = 0 \,; \quad \mathbf{A} \cdot \mathbf{X} \cdot \mathbf{T} \cdot \overline{\mathbf{X}}^t = \mathbf{I}$$

Here $\mathbf{X} = |\mathbf{x}_1, ..., \mathbf{x}_n, \overline{\mathbf{x}_1}, ..., \overline{\mathbf{x}_n}|$ is the matrix of dimension $(n \times 2n)$, $\mathbf{T} = \mathrm{diag}|\mathrm{p}_1, ..., \mathrm{p}_n, \overline{\mathrm{p}_1}, ..., \overline{\mathrm{p}_n}|$ – diagonal matrix of dimensions $(2n \times 2n)$.

3. THE EFFECTS OF EXCITATION DUE TO REAL SEISMIC EVENTS

As the analysis of the records of real seismic events reveals, the random process of the motion in foundations can be described in the form of (2.2), (2.3). In the case of large constructions like hanging bridges, the time shift, caused by finite velocity of the propagation of seismic waves, should be respected.

In most seismic events the spectral density of its stationary part has a characteristic shape: It contains one dominant frequency ω_0, nonzero value in $\omega = 0$ and fast monotone decrease for $\omega > \omega_0$. Matrix $\mathrm{S}_v(\omega)$ can be written in the form of (see Fig. 2)

$$\mathrm{S}_v(\omega) = \mathrm{S}_0 \psi(\omega) \quad ; \qquad \psi(\omega) = \frac{2\sigma_0^2}{\pi} \frac{a^2 b}{(a^2 - \omega^2)^2 + 4b^2\omega^2} \,; \qquad (a^2 > 2b^2) \tag{19}$$

where

S_0 – square symmetrical matrix of nondimensional real numbers; it's diagonal elements express the exposition of the corresponding support of the construction to the excitation. Nondiagonal elements express the degree of entire mutual correlation of excitations in two supports. Typical case is $S_{0ii} = 1$; $S_{0ij} < 1$ for $i \neq j$.

σ_0^2 – dispersion of stationary part of excitation; a final value of dispersion in the i-th support is given by product $\sigma_0^2 \cdot S_{0ii}$

a, b — constants, which determine the shape of the function $\psi(\omega)$.

The excitation process can be considered as a narrow-band one, the bandwidth of which being given by the ratio a/b; the function $\psi(\omega)$ according to (19) has the maxima in $\omega = \pm\sqrt{a^2 - 2b^2}$.

The modulation of each of the components of excitation process $\mathbf{v}(t)$ can be with acceptable accuracy formulated in the form of difference of two exponential function, see e.g.[1,4]. The fact that the excitation in individual supports starts in different moments we introduce using the Heaviside function $h(t)$, the step of which is placed in the starting moment of the excitation.

$$m_i(t) = h(t-\Delta t_i)(\mathrm{e}^{-\alpha(t-\Delta t_i)} - \mathrm{e}^{-\beta(t-\Delta t_i)}) \; ; \qquad \alpha < \beta \tag{20}$$

$$\dot{m}_i(t) = h(t-\Delta t_i)(-\alpha\,\mathrm{e}^{-\alpha(t-\Delta t_i)} + \beta\,\mathrm{e}^{-\beta(t-\Delta t_i)}) + \\ + \delta(t-\Delta t_i)(\mathrm{e}^{-\alpha(t-\Delta t_i)} - \mathrm{e}^{-\beta(t-\Delta t_i)}) ; \qquad \alpha < \beta .$$

Supposing spectral density (19) and modulation functions (20) we can simplify the expression (15). It can be rearranged in

$$\mathbf{D}_u(t) = \sum_{i,j=1}^{m} \sum_{k,l=1}^{n} \int_{-\infty}^{\infty} \int_{\Delta t_i}^{t} \int_{\Delta t_j}^{t} \sum_{\substack{k^*=k,\bar{k} \\ l^*=l,\bar{l}}} \mathbf{S}_{l^*}\, \mathrm{e}^{p_{l^*} t - (p_{l^*} - \mathrm{i}\,\omega)\tau_1} \, \cdot$$

$$\cdot\, \mathbf{S}_{mij}(\omega, \tau_1, \tau_2) \cdot \mathbf{S}^t_{k^*}\, \mathrm{e}^{p_{k^*} t - (p_{k^*} + \mathrm{i}\,\omega)\tau_2}\, \mathrm{d}\tau_1 \mathrm{d}\tau_2\, \mathrm{d}\omega \tag{21}$$

(index with bar means complex conjugate values, that is e.g. $p_{l^*} = p_l$ for $l^* = l$ and $p_{l^*} = \overline{p_l}$ for $l^* = \bar{l}$). Both integrals, in τ_1, τ_2 domains, can be simply expressed due to the form of the matrices $\mathbf{S}_{mij}$ (16), the integral in ω can be solved using the residual theorem. Thus we obtain

$$\mathbf{D}_u(t) =$$

$$= \sum_{i,j=1}^{m} \sum_{k,l=1}^{n} \sum_{\substack{k^*=k,\bar{k} \\ l^*=l,\bar{l}}} A^{FF}_{l^* ijk^*} \left[\mathcal{F}^{\phi\phi 0}_{l^* ijk^*}(t) - \mathrm{i}\,\mathcal{F}^{\phi\chi 1}_{l^* ijk^*}(t) + \mathrm{i}\,\mathcal{F}^{\chi\phi 1}_{l^* ijk^*}(t) + \mathcal{F}^{\chi\chi 2}_{l^* ijk^*}(t) \right] + \\ + A^{FG}_{l^* ijk^*} \left[\mathcal{F}^{\phi\chi 0}_{l^* ijk^*}(t) + \mathrm{i}\,\mathcal{F}^{\chi\chi 1}_{l^* ijk^*}(t) \right] + A^{GF}_{l^* ijk^*} \left[\mathcal{F}^{\chi\phi 0}_{l^* ijk^*}(t) - \mathrm{i}\,\mathcal{F}^{\chi\chi 1}_{l^* ijk^*}(t) \right] + \\ + A^{GG}_{l^* ijk^*}\, \mathcal{F}^{\chi\chi 0}_{l^* ijk^*}(t) \tag{22}$$

where

$$A^{XY}_{lijk} = s_{ij} \mathbf{S}_l \mathbf{X}_i \mathbf{Y}^t_j \mathbf{S}_k \, , \quad \mathbf{X}, \mathbf{Y} \in \{\mathbf{F}, \mathbf{G}\} \, , \quad (\mathbf{X}, \mathbf{Y} \text{ can be either } \mathbf{F} \text{ or } \mathbf{G})$$

$$\phi_{li}(t,\omega) = \int_{\Delta t_i}^{t} \mathrm{e}^{p_l t - (p_l - \mathrm{i}\,\omega)\tau}\, \dot{m}_i(\tau)\, \mathrm{d}\tau \; ; \qquad \chi_{li}(t,\omega) = \int_{\Delta t_i}^{t} \mathrm{e}^{p_l t - (p_l - \mathrm{i}\,\omega)\tau}\, m_i(\tau)\, \mathrm{d}\tau$$

$$\mathcal{F}^{\xi\eta r}_{lijk}(t) = \int_{-\infty}^{\infty} \omega^r \xi_{li}(t,\omega) \psi(\omega) \eta_{kj}(t,-\omega)\, \mathrm{d}\omega \, , \quad \xi, \eta \in \{\phi, \chi\} \, . \tag{23}$$

The residual theorem is applied to integral (23).

4. NUMERICAL EXAMPLE

Let's analyse the horizontal seismic response of a bridge, which has been modelled as a symetric simple beam with 5 lumped masses $m_1 \div m_5$, on elastic supports (high piers, spring constants $K_1 = K_2 = K_{sup}$), see Fig. 1. The elastic constants are C^{el}_{ik}, being defined as forces in the points k, which cause unitary displacement of the beam in the point $i \equiv k$ and zero displacement in other points $i \neq k$.

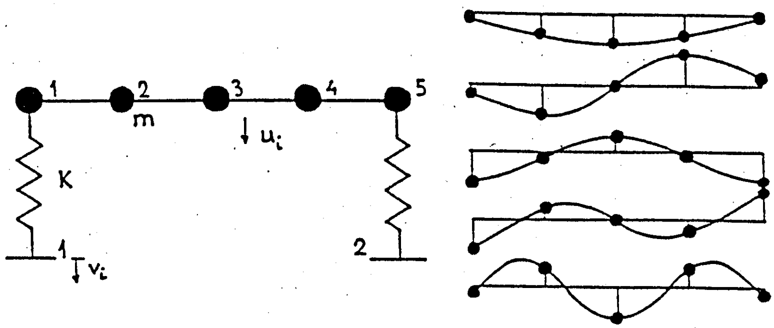

Fig. 1 Scheme of the structure, its natural modes.
$\omega_1 \div \omega_5$= 6.508, 24.840, 37.680, 41.610, 63.515 rad/sec.

The seismic excitation is being transferred into the structure by means of the rigidity and damping of the supports $K_{sup} = 1.77\text{E}9$ N/m, $B_{sup} = 0.98\text{E}7$ N.s/m, as defined by the matrices on the right-hand side of equ. (1)

$$\mathbf{G} = \begin{bmatrix} 1.77E9 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1.77E9 \end{bmatrix} ; \quad \mathbf{F} = \begin{bmatrix} 0.98E7 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.98E7 \end{bmatrix}$$

The matrix **A** in (2.1) is a diagonal one, containing the masses m_i on its diagonal. **B** is diagonal too, the members of the matrix **C** are equal to spring constants of the beam C^{el}_{ik} with the exception of the supports, where the "external" springs influence on the motion of the masses in the supports. We have therefore

$$c_{ik} = C^{el}_{ik}, \text{ with the exception of } \quad c_{11} = c_{55} = C^{el}_{11} + K_{sup}.$$

Thus we obtain

$$\mathbf{A} = \begin{bmatrix} 1.24\text{E}6 & 0 & 0 & 0 & 0 \\ 0 & 1.15\text{E}6 & 0 & 0 & 0 \\ 0 & 0 & 9.0\text{E}6 & 0 & 0 \\ \vdots & & & & \end{bmatrix} \quad \text{kg}$$

$$\mathbf{B} = \begin{bmatrix} 1.47\text{E}7 & 0 & 0 & 0 & 0 \\ 0 & 2.85\text{E}6 & 0 & 0 & 0 \\ 0 & 0 & 5.90\text{E}5 & 0 & 0 \\ \vdots & & & & \end{bmatrix} \quad \text{N.s/m}$$

$$\mathrm{C} = \begin{bmatrix} 2.00E9 & -5.20E8 & 3.67E8 & -9.17E7 & 1.53E7 \\ -5.20E8 & 1.41E9 & -1.35E9 & 5.50E8 & -9.17E7 \\ 3.67E8 & -1.35E9 & 1.95E9 & -1.35E9 & 3.67E8 \\ \vdots & & & & \end{bmatrix} \quad \mathrm{N/m}$$

From these given characteristics of the structure the complex eigenvalues p_l and eigenvectors $\mathbf{x}_l$ have to be calculated, as the roots of (18). We obtain e.g. (dimension 1/sec):

$$p_1 = -0.854 + 6.453\ \mathrm{i}\,;\ p_2 = -1.788 + 24.878\ \mathrm{i}\,;\ p_3 = -5.518 + 37.324\ \mathrm{i}\,;\ \ldots$$

The first two eigenvectors (dimensionless) are

$$\mathbf{x}_1 = \begin{bmatrix} 1.00 & 0.00\ \mathrm{i} \\ 19.32 & 1.20\ \mathrm{i} \\ 26.89 & 1.79\ \mathrm{i} \\ 19.32 & 1.20\ \mathrm{i} \\ 1.00 & 0.00\ \mathrm{i} \end{bmatrix}\ ; \quad \mathbf{x}_2 = \begin{bmatrix} 1.00 & 0.00\ \mathrm{i} \\ 2.79 & 0.59\ \mathrm{i} \\ 0.00 & 0.00\ \mathrm{i} \\ -2.79 & -0.59\ \mathrm{i} \\ -1.00 & 0.00\ \mathrm{i} \end{bmatrix}$$

From these eigenvectors we obtain the matrices $\mathbf{S}_l$ using the relation (17). These matrices, here of the order 5x5, are complex, symetrical and those, corresponding to complex coniugate roots, are complex coniugate. They have the meaning of coefficients for the decomposition of the excitation (12), as explained in [7,8]; the decomposition itself reminds the decomposition of harmonic loading into natural modes.

Lets consider the input signal with the stationary random part of the type (19) with the peaks near the 1st and 2nd natural frequencies of the structure. The modulation function be of the type (3.2) with its maximum value at $t = 1.20$ sec, the excitation of the second support starts with the time delays of zero, half and total of the prevailing period of the excitation (see Fig. 2).

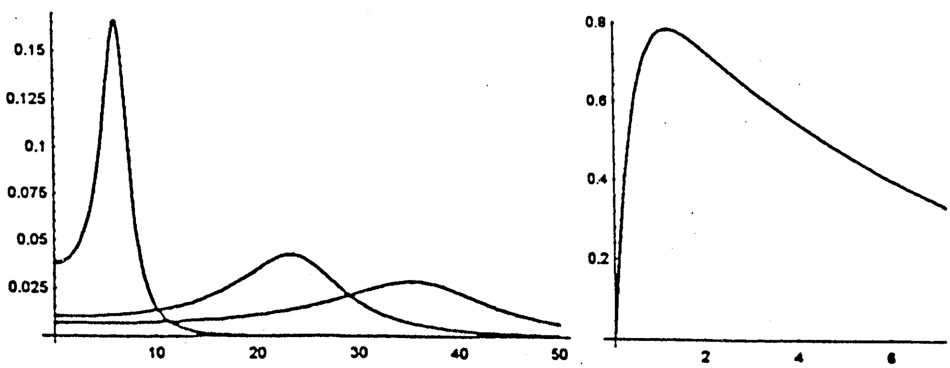

Fig. 2 The excitation (3.1): $\sigma_0 = 0.90$ m, $(a, b) = (6.5,1.6)$; $(24.8,6.2)$ rad/sec.
The modulation function (3.2): $\alpha = 0.15$, $\beta = 2.5$,
the time-shifts $\Delta t = (0, 0.48, 0.97)$; $(0, 0.13, 0.25)$ sec.

The dispersions of the response of the masses $m_1 \div m_5$ are time-dependent, due to the nonstationarity of the process. They were calculated from the equ. (15), which was rearranged into (22) and (23). The resulting graphs of the dispersions (i.e. the absolute values of the diagonal members of the matrix $\mathbf{D}_u(t)$) plotted against the time are on the Fig. 3.

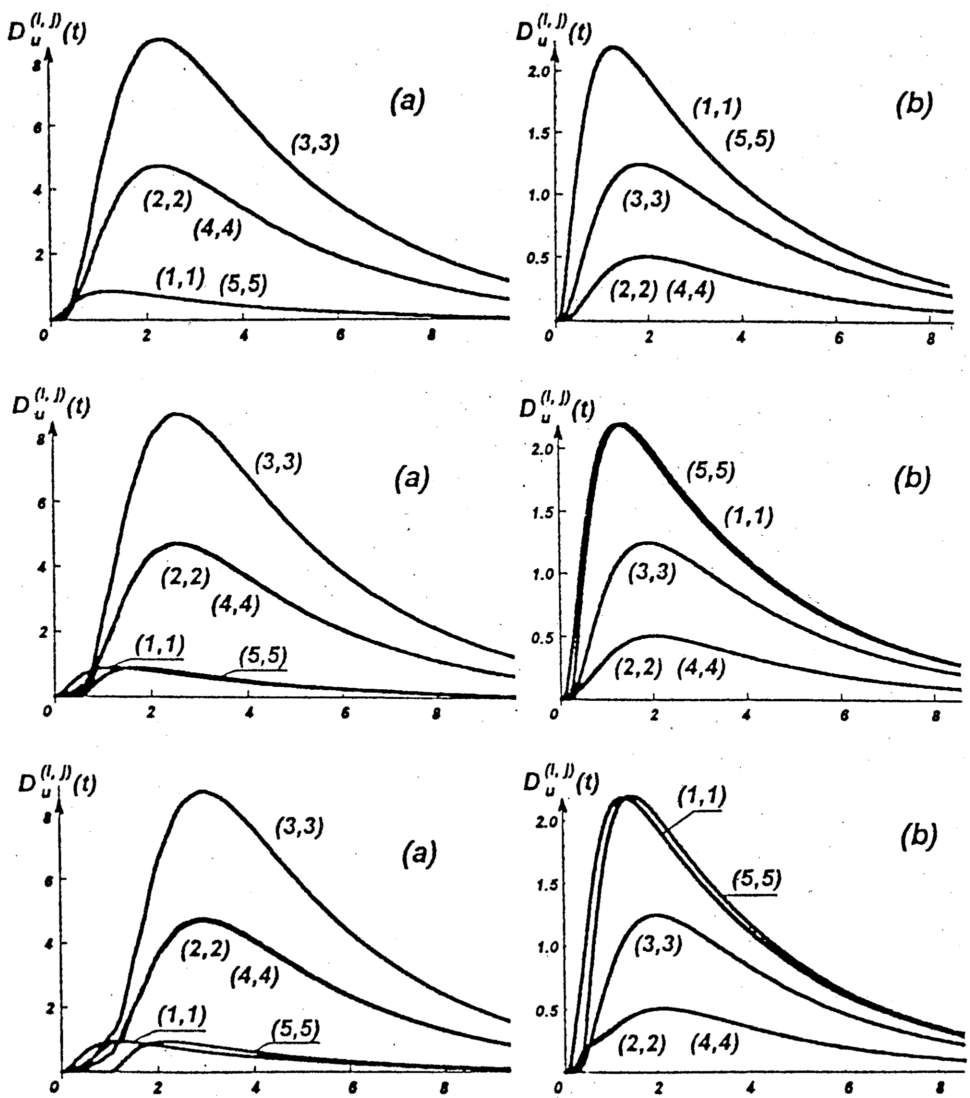

Fig. 3 Dispersions of the masses $1 \div 5$ as functions of time.
Case a): the excitation with the prevailing frequency $\omega_0 = 6.50$ rad/sec, $\Delta t = 0.0, 0.48, 0.97$ sec.
Case b): the excitation $\omega_0 = 24.8$ rad/sec, $\Delta t = 0.0,\ 0.13,\ 0.25$ sec.

5. CONCLUSIONS

The resulting graphs on Fig. 3 show a quite clear diversity between the responses of symetric masses 2,4 or 1,5 for greater time-delays of the excitation in both supports. A significant difference between the magnitude of the response has not been noticed for the considered parameters of the excitation, a deeper parametric analysis being forseen for next future.

Nevertheless the presented extension of the previously described method [7,8] has proved itself as useful, as it makes possible to respect the nonstationarity as well as the difference in the excitations in distant supports of the structure in stochastic terms, solving the problem analytically. The numerical realization of the method is in principle practicable, of course with some exigencies to the software and to the computer time (about 30 min. for one of the 6 graphs on Fig. 3, using PC 486).

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